What Are “Learning Trajectories” and How Do They Help?

Douglas H. Clements and Julie Sarama
University of Denver, Morgridge College of Education

What approaches show promise for teaching young children challenging but achievable mathematics? Recently, educators have developed and used learning trajectories. As students learn about a mathematical topic, they progress through increasingly sophisticated levels of thinking. At each level, children can solve new types of problem. These levels form a developmental progression—the basic skeleton of a learning trajectory. Contributors to the Common Core (CC, CCSSO/NGA, 2010) began by writing concise developmental sequences learning trajectories for many major topic of mathematics. The three major writers then sliced these learning trajectories into grade-level specific standards.

A complete learning trajectory has three parts: a goal, a developmental progression of levels of thinking, and associated instructional activities. To attain a certain mathematical competence in a given topic or domain (the goal), students progress through several levels of thinking (the developmental progression), aided by tasks and experiences (the instructional activities) specifically designed to help students build the next level of thinking (Clements & Sarama, 2004).

Let's use an example from counting. Our goal will be competence in object counting with understanding (the mathematics may seem simple, but is surprisingly complex; see Clements & Sarama, 2009; National Research Council, 2009).

The developmental progression covers many years in children’s lives and has many levels (see Clements & Sarama, 2009; National Research Council, 2009). Here we
will describe only a couple. First, children learn simple verbal counting. Even here there are levels, not only in the number reached, but the order and separation of words (i.e., moving beyond “one, two, three, four, five, six, seven”). Children then learn to maintain 1-to-1 correspondence between these counting words and objects. At first they can do this only with a limited number of objects (5 or fewer) laid in a straight line.

To check the next level, we ask children, "How many are there?" Children who are only at the previous, “corresponder,” level will often recount the objects rather than answering "five." On the other hand, if the child quickly and confidently answers "five," she is at the level of understanding cardinality. We might lay out more objects, and eventually lay out objects not structured into a straight line to check higher levels (see Clements & Sarama, 2009).

Let us now consider the third part of a learning trajectory, the instructional tasks. For example, say a boy was found to be at the corresponder level only, unable to answer the "how many?" question. Our learning trajectory describes a research-based strategy that will help him connect his existing skills in counting with his ability to subitize—to recognize the number in a very small collection quickly without counting (this is a skill our Building Blocks curriculum focuses on early and consistently; see Clements & Sarama, 2013). Because cardinality is built into subitizing, combining the two processes—subitizing and counting—is effective in encouraging the development of cardinality. The activity *How Many?* builds, connects, and applies these competencies. The teacher tells children she has placed as many cubes (say 4, hidden) in her hand as she can hold. She asks children to count with her to see how many. She then takes out one at a time as the group says the number word. She repeats the last counting number, “four,”
gesturing in a circular motion to all the cubes, and says, “That’s how many I could hold.”

She then challenges children to see how many they can hold during playtime.

Notice how, as children count with the teacher who shows the blocks one at a time, saying the number word aloud, they see (i.e., subitize) one when they say “one,” they see two when they say "two," and so forth. This can imbue counting with quantitative (cardinal) meaning. At the end, repeating the last counting number, “three,” gesturing in a circular motion to all the cubes, and saying “That’s how many there are in all” emphasize that cardinal meaning. Many other activities could be used, but research suggests that activities that similarly connect students’ counting and subitizing will be particularly effective.

In these ways, learning trajectories can support children’s learning, as well as assessment, curriculum development, and teaching. Many other resources for learning trajectories are available (e.g., Clements & Sarama, 2009, 2013; Confrey, Maloney, & Nguyen, in press; National Research Council, 2009; Sarama & Clements, 2009). Children whose teachers use research-based learning trajectories demonstrate higher levels of mathematical reasoning. Current research in learning trajectories points the way toward mathematics that is more effective and efficient, but still creative and enjoyable.
References


