Objectives: To introduce finding the volume of irregular objects using a water displacement method.

**Key Concepts and Skills**
- Use tables to record data. [Data and Chance Goal 1]
- Use a displacement method to find the volume of irregular solids. [Measurement and Reference Frames Goal 2]
- Calibrate and use a metric measuring tool to compare volume and capacity. [Measurement and Reference Frames Goal 2]

**Key Activities**
Students calibrate a bottle and use it to measure the volume of various objects by a displacement method.

**Materials**
- Math Journal 2, pp. 382 and 383
- Study Link 11-4
- Slate, dictionary, per workstation: ruler, measuring cup, tape, paper towels, 2-L plastic bottle (top cut off), paper, scissors, rubber bands, large can or jar filled with 2 L of water, rocks and other objects for displacement activities

**Advance Preparation**
For Part 1, make a model of a calibrated bottle (journal page 382). Organize workstations for groups of four students. Furnish each workstation with the materials listed on journal page 382. Cut off the top part of each 2-L bottle, about 9 in. from the bottom. Each station requires an unopened can of a nondiet soft drink (diet drinks will float), rubber bands, several rocks, and about 4 other objects whose volume can be measured. You will also need a supply of paper towels. If you do not have easy access to a sink, use a bucket of water. For the optional Readiness activity in Part 3, obtain a copy of the book *Who Sank the Boat?* by Pamela Allen (Paperstar, 1996). Equip a workstation with a bucket or clear container half-filled with water, an empty margarine tub or yogurt container, rocks, waterproof marker. Students explore the principle of displacement by solving a thought experiment about a boat and a stone.

**Solving American Tour Problems**
- Math Journal 2, p. 384
- Student Reference Book, pp. 338–395
  - Students solve problems using American Tour data.

**Solving Additive Volume Problems**
- Math Journal 2, pp. 384A and 384B
  - Students solve real-world problems involving the volumes of two adjacent rectangular prisms.

**Math Boxes 11-5**
- Math Journal 2, p. 385
  - cm ruler or Geometry Template
  - Students practice and maintain skills through Math Box problems.

**Study Link 11-5**
- Math Masters, p. 337
  - Students practice and maintain skills through Study Link activities.

**REACHENMENT**

**Finding Volume by Displacement**
- Math Masters, pp. 338 and 339
  - per workstation: bucket or clear container (partially filled with water), empty margarine tub or yogurt container, rocks, waterproof marker
  - Students explore the principle of displacement by solving a thought experiment about a boat and a stone.
How to Calibrate a Bottle

**LESSON 11**

**Materials**
- 2-L plastic soft-drink bottle with the top cut off
- can or jar filled with about 2 L of water
- measuring cup
- ruler
- scissors
- paper
- tape

1. Fill the bottle with about 5 inches of water.
2. Cut a 1 in. by 6 in. strip of paper. Tape the strip to the outside of the bottle with one end at the bottle top and the other end below the water level.
3. Mark the paper strip at the water level. Write "0 mL" next to the mark.
4. Pour 100 milliliters of water into a measuring cup. Pour the water into the bottle. Mark the new water level, and write "100 mL."
5. Pour another 100 milliliters of water into the measuring cup. Pour it into the bottle, and mark the new water level "200 mL."
6. Repeat, adding 100 milliliters at a time until the bottle is filled to within an inch of the top.
7. Pour out the water until the water level in the bottle falls to the 0 mL mark.

How would you use your calibrated bottle to find the volume of a rock?

Add water 100 mL at a time.

- Fill the bottle with water to the 0 mL mark. Drop the rock into the bottle. The amount that the water rises shows the volume of the rock in milliliters.
Find the volumes of several other objects. For example, find the volume of a

Math Journal 2, LESSON 17

Does it matter whether you make

What is the new level of the water in the bottle? mL

until water reaches the rubber band.

0-mL level. Place a rubber band
as your heart. Here is a way to find

Water level at 200 mL

Volume of Water
100 mL
500 mL
200 mL
300 mL
No
Answers vary.

Demonstrating a Displacement Method for Finding the Volume of an Irregular Object

Science Link Use your calibrated bottle to demonstrate a
displacement method. Fill the bottle with water to the 0-mL
level, and drop a rock into the bottle. The water will rise to a
higher level. This shows the volume of the rock in milliliters. (See
margin.) To support English language learners, explain the
meaning of displacement in this context, and use the vocabulary to
describe what is happening with the water.

Remind students that 1 cubic centimeter is equal to 1 milliliter. Ask the following questions:

• How many cubic centimeters are equal to 200 mL? 200 cm³
• How many milliliters are equal to 1 L? 1,000 mL
• How many cubic centimeters are equal to \( \frac{1}{2} \) L? 500 cm³, or \( \frac{1}{2} \) L = 500 mL = 500 cm³
• What is the volume of the demonstration rock in cubic centimeters? It is the same as the volume in milliliters.

NOTE The volumes of most solids, such as rocks, are usually reported in cubic units. The volumes of liquids are usually reported in units such as liters, milliliters, gallons, quarts, and pints.

Make sure students understand why this method can be used to
find the volume of an object. Discuss the following points:

• Water has volume. When an object is added to the water, it
takes up (displaces) space that was previously occupied by
water. This pushes the water level higher than it was. The
volume of water between the original level mark and the new
level mark must be equal to the volume of the added object. (If the object floats, it must be forced down so it is completely submerged.)

You don’t need to know the volume of the water in the container before the object is added—only the change in water level. The paper scale on the calibrated bottle allows you to read the change in water level directly.

### Using a Calibrated Bottle to Measure the Volumes of Various Objects

*(Math Journal 2, p. 383)*

Each group will need several rocks and about four other objects. If the number of objects is limited, groups should trade objects. Have students use the 100-mL intervals on the paper scale to estimate the water level to the nearest 10 mL or at least to the nearest 25 mL after adding an object to the bottle. Assign groups to complete Problems 1 and 2 on journal page 383. Circulate and assist.

When groups have finished, discuss the last question in each of the problems. Explain that volume is conserved—that is, it remains unchanged under different arrangements of the material.

Assign Problem 3. Have students begin with an unopened can of a nondiet soft drink. They can check their volume estimates (in milliliters) against the can label.

**NOTE** A 12-oz can of regular cola contains about 10 tsp of sugar. Because the molecules of sugar spread evenly in the empty spaces between the water molecules, the sugar dissolves in the liquid without increasing its volume. Therefore, the additional molecules of sugar in the cola make it more dense. Diet colas are usually sweetened with aspartame, which is 160 times sweeter than sugar. Considerably less aspartame is needed to sweeten diet cola; therefore, a can of diet cola weighs less than a can of regular cola, is less dense, and is more likely to float.

If any groups work with the same kind of small objects, such as golf balls, they should estimate the combined volume for 4 or 5 balls and then divide the result by the number of balls. (One golf ball has a volume of about 41 cm³.) This will yield more accurate results than estimating the volume of a single ball because the measurement error for a small object is likely to be a larger percent of the measurement than the measurement error for a larger object.

### Ongoing Learning & Practice

#### Solving American Tour Problems

*(Math Journal 2, p. 384; Student Reference Book, pp. 338–395)*

Students solve problems using data from the American Tour section of the *Student Reference Book.*
Solving Additive Volume Problems

(Math Journal 2, pp. 384A and 384B)

Students solve real-world problems involving the volumes of two adjacent rectangular prisms. Then they add the two volumes to find the volume of the entire solid. Review the two formulas used to find the volume of prisms: \( V = B \times h \) and \( V = l \times w \times h \). When most students have finished, ask volunteers to share their solutions.

Math Boxes 11-5

(Math Journal 2, p. 385)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 11-7. The skill in Problem 5 previews Unit 12 content.

Writing/Reasoning Have students write a response to the following: Explain how you found the simplest form of \( \frac{29}{3} \) in Problem 1. First I renamed \( \frac{29}{3} \) as a mixed number by dividing 29 by 3. \( \frac{29}{3} = \frac{9\frac{2}{3}}{3} \). A mixed number is in simplest form if the fraction part is in simplest form. In \( 9\frac{2}{3} \), the fraction \( \frac{2}{3} \) is in simplest form because the numerator and the denominator cannot be divided by a common factor greater than 1.

Study Link 11-5

(Math Masters, p. 337)

Home Connection Students perform a displacement experiment. You may choose to have them do this during class time or offer it as an optional home assignment.

3 Differentiation Options

Reading about Displacement

Literature Link To explore the concept of displacement, have students tell which character they think is responsible for the mishap in the book Who Sank the Boat? Read the book and have students predict which character displaced so much water that the boat finally sank. Share the illustrations and discuss how characters balanced each other in the boat. Then discuss the results of the story.

Who Sank the Boat?

Summary: As each character gets into the boat, the waterline approaches the rim of the boat. Surprisingly, the smallest creature of all is the one who causes the boat to sink.
LESSON 11

A Boat and a Stone

A thought experiment uses the imagination to solve a problem. Mathematicians, physicists, philosophers, and others use thought experiments to investigate ideas about nature and the universe.

One early example of a thought experiment attempts to show that space is infinite. Use your imagination to picture what is being described in the experiment below.

If there is a boundary to the universe, we can toss a spear at it. If the spear flies through, it isn’t a boundary after all. If the spear bounces back, then there must be something beyond the supposed edge of space—a cosmic wall which is itself in space that stopped the spear. Either way, there is no edge of the universe; space is infinite.

Often it is impossible to investigate the situation in a thought experiment directly. This might be because of physical or technological limitations. But the thought experiment in Problem 1 can be modeled directly. Solve Problem 1, and then follow the directions in Problem 2 to model the experiment.

1. Imagine that you are in a small boat. There is a large stone in the bottom of the boat. The boat is floating in a swimming pool. If you throw the stone overboard, does the level of the boat on the water go up, down, or stay the same? Does the level of the water in the pool go up, down, or stay the same?

   The boat will go up. The water level of the pool will remain the same.

2. Model the thought experiment, “A Boat and a Stone.”

   Materials
   - bucket or clear container
   - small container that floats and fits in the bucket or clear container with plenty of space all around
   - several rocks
   - water
   - waterproof marker

   Directions:
   a. Fill the bucket part way up with water. Make sure the water is deep enough to cover the rocks.
   b. Place a rock in the small container, and float it in the bucket. If the small container sinks, try a smaller rock. If the small container tilts over into the water, try a larger rock.
   c. After the water settles, mark the height of the water on the outside of the bucket with the marker. If the bucket is clear, mark the inside wall. Also, mark the height of the water on the outside of the small container.
   d. Take the rock out of the small container, and gently drop it into the water.

   The water level on the container was lower.

   e. Describe the changes in the height of the water in the bucket.

   The water level on the container was lower after I picked up the rock, then it rose after I dropped the rock in the water.

   f. Do the changes agree with your thought experiment solutions? Why or why not?

   Answers vary.

STUDY LINK

A Displacement Experiment

Try this experiment at home.

Materials
   - drinking glass
   - water
   - 2 large handfuls of cotton

   (Be sure to use real cotton. Synthetic materials will not work.)

Directions
   a. Fill the drinking glass almost to the top with water.
   b. Put the cotton, bit by bit, into the glass. Pull it up as you go.

   If you are careful, you should be able to fill all of the cotton into the glass without spilling a drop of water.

   Think about what you know about displacement and volume. Why do you think you were able to fill the cotton into the glass without overflowing?

   Most of the space taken up by a handful of cotton is air between the fibers, so it did not displace too much water.

Teaching Master

Name: ___________________ Date: ___________ Time: ___________

LESSON 11

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   - water
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   Directions:
   a. Fill the bucket part way up with water. Make sure the water is deep enough to cover the rocks.
   b. Place a rock in the small container, and float it in the bucket. If the small container sinks, try a smaller rock. If the small container tilts over into the water, try a larger rock.
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   Answers vary.

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